C.U.SHAH SCIENCE COLLEGE, AHMEDABAD

SEMESTER-IV (MATHEMATICS)

INTERNAL EXAMINATION , MAT-205

Date :- 18 /03/2015

Marks:50

INSTRUCTION: Write the answer of quiz on **first** page of Answer sheet.

Q-1 Suppose G be a group and a and b are elements of G with $ab = ba$ then prove that $ab^n = b^n a$; for each $n \in N$.

OR

Q-1 Prove that an element [m] in Z_n has a multiplicative inverse iff (m,n) = 1.

Q-2 Define group Prove that the set G of rational numbers other than 1 with operation $\oplus$ such that $a \oplus b = a + b - ab$ for $a, b \in G$ is an abelian group.

OR

Q-2 Define : Subgroup, Index.
State and prove Lagrange's theorem for finite group.

Q-3 Define Normal subgroup. Prove that a subgroup H of G is a normal subgroup of G if and only if $aHa^{-1} \subset H$ for each $a \in G$.

OR

Q-3 Give an example of a non-commutative group G , each of whose subgroups is normal.

Q-4 Define Kernel of a homomorphism between two groups. Prove that the kernel of a homomorphism is a normal subgroup.

OR

Q-4 For the cyclic group $G = \{e, a, a^2, a^3, ..., a^{23} / a^{24} = e\}$ of order 24

Obtain (i) Orders of subgroups generated by a^8, a^{11}, a^{12} and
(ii) All the generators of G.

QUIZ

Answer the following questions in short.

(1) In Z_7, the congruence class modulo 7,the inverse of [5] is ----
(a) [1] (b) [3] (c) [5] (d) does not exist

(2) $\phi(n)$ is an even for n= ______
(a) 1 (b) 2 (c) 3 (d) none of these

(3) For a,b, c∈ G ,where G is a group; $(abc)^{-1} = ---$
(a) a b c (b) $a^{-1}b^{-1}c^{-1}$ (c) $c^{-1}b^{-1}a^{-1}$ (d) none of these

(4) The set G of all real numbers except -1 forms a group under the binary operation * defined by a*b= a + b + a b . The identity element of this group is ----
(a) 1 (b) 2 (c) 0 (d) none of these

(5) The number of right cosets of any subgroup of a finite group is ------- the number of left cosets.
(a) greater than (b) equal to (c) less than (d) none of these

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